



Students' Active Engagement in Inquiry Mathematics Classroom

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Abstract

The purpose of this study is to introduce Inquiry Teaching in the classroom and to enable students' active participation in learning. Kemmis, McTaggart & Retallick's (2004) model of Action Research was used with the dual role of teacher and researcher with Class Six students in a private school of Karachi, Pakistan. Fink's (1999) model of Active Learning was used to engage students in the classroom and according to this model, active learning means to do, discuss and observe (D-D-O) Mathematics. Data was generated through observations, informal conversations with students and a teacher along with students' samples of work and field notes. Some of the important findings and conclusions that are drawn are that inquiry teaching based on D-D-O may have changed the role of a teacher from a dictator to a facilitator and a participant in the process of learning. In addition, Students involved in the classroom through D-D-O need to unlearn some old practices to become decision-makers in the learning process. Furthermore, Inquiry through D-D-O model enables the students to make connections between Mathematical ideas and to understand the nature of learning Mathematics and open-ended task. These elements and strategies may be laborious but we should initiate a change process in our classroom because a journey of thousand miles begins with a single step.

Key Words: D-D-O Frame work, Inquiry teaching, learning, student's active participation, active learning.

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Introduction

This study is an effort to bring change in the classroom environment by making the classroom more student-centered and student-friendly through the implementation of inquiry teaching in the lower secondary Mathematics classroom.

The reform movements in Mathematics education comprising Mathematics teachers, educators and researchers came to the conclusion that Mathematics is a socially constructed domain and it can be learned through active student participation in a Mathematics classroom (Borasi, 1992; Jaworski, 2006; Klein, 2004).

The next step of the reform was to introduce a teaching strategy that can address mathematical learning based on active student participation. Research in the field of Mathematics revealed that inquiry is one of the teaching strategies which can help students to construct their own mathematical meaning based on their prior ideas and experiences (Borasi, 1992; Jaworski, 2006; Klein, 2004). In addition, Mathematics teachers, educators and researchers through research came to the conclusion that inquiry is a teaching and learning

process in which students can involve themselves actively in meaning-making, discussion and investigation (Anderson, 2000; Hogan & Berkowitz, 2000), which is needed to become an independent learner and decision maker in the process of knowledge construction.

Inquiry and Inquiry Teaching

The world of inquiry, curiosity, and wonder should be alive in a classroom everywhere (Plowright & Watkins, 2004). Inquiry and inquiry teaching is defined by different people differently and there is no common definition of inquiry that can be accepted universally. For example, Colburn (2000) has defined it as, “The creation of a classroom where students are engaged in essentially open-ended, student-centered, hands-on activities” (p.42). In addition, children are naturally curious to know and inquiry supports the way children are thinking and giving them specific direction to explore new areas (Joyce, Weil, & Calhoun, 2000). Furthermore, Jaworski (2007) defined inquiry, as “To inquire means to ask a question, to make an investigation, to acquire information or to search for knowledge” (p.5).

The use of inquiry teaching in Mathematics is needed due to the nature of the subject which is progressive and fallible, thus an incomplete work in progress (Ernest, 1999) and progression could be made possible through active interaction with the subject (Klein, 2004; Wilhelm & Walters, 2006). In addition, Ernest (1999) with reference to the book *Social Constructivism as a Philosophy of Mathematics* says that Mathematics is not only fallible but is created by a group of people through active interaction with the subject. In addition, Mathematics is not like an objective entity that can be transferred from one individual to another rather it is constructed through active involvement like a professional mathematician in the Mathematics classroom (Corte, 2007).

Jaworski (2006) commented that the term inquiry in Mathematics is referred to as the active construction of knowledge through personal involvement in the construction process. Inquiry in Mathematics fits with the constructivist view of knowledge which results in conceptual and relational understanding. In inquiry teaching, prior ideas of children are given respect and importance and the new knowledge or

understanding that is built on that as a base for further construction of the knowledge. In this way, students see a connection between the ideas in their minds and the new ideas (Jacobs, 2005). In addition, it is considered that mathematical knowledge can be constructed in a community of inquiry and students' learning of Mathematics will improve as teachers come to know more about the learning process and the tools that promote learning (Nicol, 1999).

In the traditional transmission method of teaching, authority and power are mostly in the hands of the teacher, and the students' role is passively receiving the instruction of the teacher (Gulati, 2008). On the other hand, in inquiry teaching student is more empowered as compared to the teacher and is given autonomy to solve the problem in their own way through active involvement in the process of knowledge construction (Anthony, 1996; Carter, 2004; Jacobs, 2005). Borasi (1992) defined inquiry teaching, "A pedagogical approach based on inquiry and problem posing, in contrast, is likely to "empower" students as they take on greater responsibilities for their own learning and begin to

approach knowledge as a tool over which they have control and which they can use for their own ends” (p.188). Empowering means autonomous learning having no external control over learners and according to Piaget and Dewey external control on children limits intellectual and moral growth (Carter, 2004). Empowering the students in the Mathematics classroom, in other words, means actively involving them in the construction of knowledge because it is considered that passive children are not learning (Walle, 2007). In addition, Keefer (2002) commented, “The learner must come to know the world through their own activity” (p.396). Thus, we can say empowering students in the classroom means designing tasks in such a way that the students construct mathematical understanding in their own way taking responsibility for their own learning (Adler, 1997).

In a Pakistani classroom, the teacher is the sole source of authority and knowledge and the task of the student is to receive the knowledge silently and most of the teachers teach in the way taught by their teachers (Halai, 2001; Halai, 2005; Riaz, 2008). In addition, the social interaction and cooperative

learning are lacking and their potential is unexplored in a Mathematics classroom (Halai, 1998). However, some of the studies conducted in Pakistan using the constructivist approach of teaching and learning proved successful to enhance the learning outcome of the students through open-ended activities in the classroom (Halai, 2006). It has been proved through research that good teaching means active participation of the students in the classroom to construct knowledge of their own (Dean, 2009). According to Halai, (2004) new teaching strategies that are introduced in a classroom improve the learning outcome of the students. In addition, she says that students go beyond classroom and textbook to relate Mathematics with daily life and are able to find linkages between previous knowledge and new knowledge.

Inquiry and Inquiry Learning

For long decades it had been thought that knowledge is like an objective entity lying there waiting for someone to pick up and digest. The role of a learner according to this theory is like a passive recipient and knowledge can be imparted from teacher to learner through

instruction, lecture and practice (Gulati, 2008). In addition, the role of a learner is to memorize imparted facts and reproduce the same facts like a machine, which is termed as instrumental understanding (Skemp, 1991). Furthermore, according to this theory of knowledge, the mind of a child is like a blank slate (tabula rasa) and the task of a teacher is to fill the empty space (Freire, 1998).

However, several types of research in the field of education reveal that the transmission view of learning is against the nature of human beings, which is progressive and developmental in nature (Freire, 1998; Piaget, 1963).

Similarly, the static nature of human beings has also been challenged by educationists like Dewey and child psychologists like Piaget. According to Driscoll (1994), Piaget has been mentioned as a biologist philosopher and child psychologist. Piaget called himself a constructivist because according to his point of view knowledge is a process of continuous self-construction. Knowledge is not like an object lying out there to be picked up by the child rather it is constructed by the child through interaction with the external

environment. In addition, he says that knowledge is invented and constructed as the child grows and interacts with the external surroundings. Furthermore, he explained that the child acquires knowledge through action and makes meaning from their actions. Moreover, knowledge can be acquired through activities and active participation having the autonomy to the learner, which is a central tenet of inquiry learning (Borasi, 1992; Jaworski, 2006). Similarly, Dewey (2004) used a term democracy equated with autonomy in the classroom. He said democracy is desirable for society and in a similar way in a classroom situation. In a likewise manner inquiry learning is highly valued as independent and autonomous learners to construct their own meaning in the classroom. Carter (2004) quoted Dewey and Piaget saying that external control on children limits intellectual and moral growth.

Inquiry learning based on Constructivist Philosophy advocated the active participation of students in the Mathematics classroom (Anghileri, 2006; Jacobs, 2005; Jaworski, 2006; Walle, 2007). Active participation means the interaction of students with each

other and with the materials provided to make their own mathematical construction through discussion and meaning-making (Elbers, 2003). In addition, Mathematics inquiry relates to the active construction of knowledge, where ‘active’ means personal involvement (Jaworski, 2006). Personal involvement in the task allows the student to boost their own mathematical creativity and mathematical thinking through investigation and personal discourse (Goos, 2004; Bloom, 2007). In addition, active learning enables the student to grasp the conceptual meaning of Mathematics in addition to understanding how Mathematicians study Mathematics.

Active Learning

Fink (1999) has developed a model or framework for active learning which was adapted with some changes and modifications for the research work. According to Fink, active learning means active involvement of the student in experience and dialogue, which has been further divided into doing, observing and dialogue with self and others respectively. Both Borasi (1992) and Jaworski (2006) say that

Mathematics is a socially constructed domain in which students should be engaged in dialogue to construct knowledge. Hence, doing and observing (watching/listening) greatly helps the student in Mathematics to involve them actively in the classroom. Moreover, dialogue which is included in the model is an important part of inquiry learning which enables the student to discuss, argue and refute to engage in positive mathematical discourse.

The below model has the room to allow students to work collaboratively which is highly appreciated in Mathematics Education (Goos, 2004). Goos (2004) further commented referring to Vygotsky that working in collaborating peer group enables students to construct and own their ideas and to feel like active members with the peer constructing mathematical insights through active participation. Furthermore, working collaboratively with peers not only enhances social interaction among the students rather it enhances the learning outcome of the students through active interaction with peers. McGregor (2007) argued that “The extent to which peers interact to collaboratively work on problems

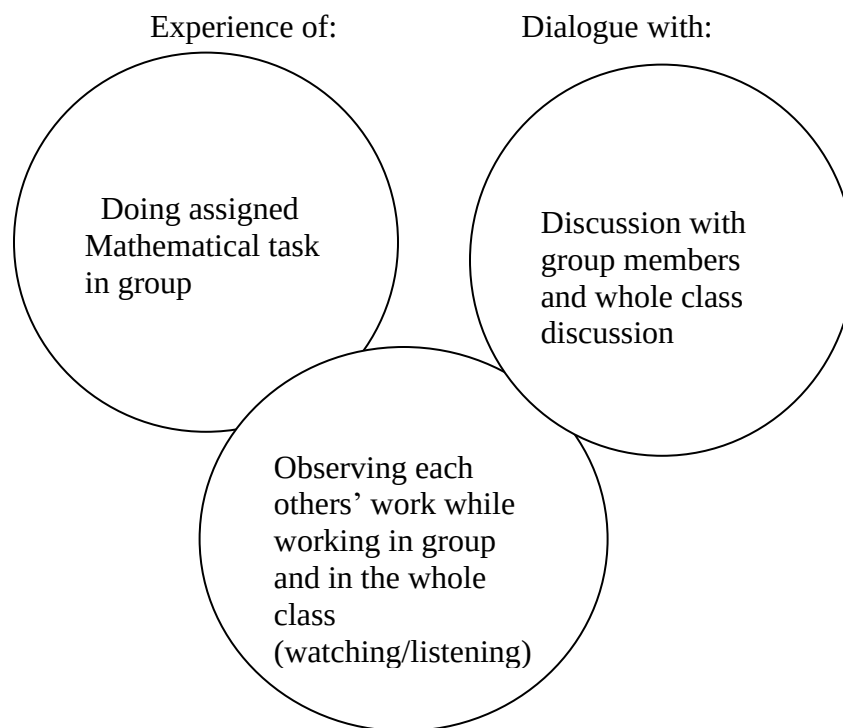


Fig.1, *A Model of Active Learning (Fink, 1999)*

together, it influences the thinking process engaged in and the subsequent learning outcomes” (p.56).

Methods and Procedures

Action Research is mostly suitable for those inquirers who want to bring change in classroom practices and activities through action and reflection (Costello, 2003; Ferrance, 2000; Kemmis, McTaggart & Retallick, 2004; Koshy, 2005; Skerritt, 1992). In addition, Dean (2009) says, “Action Research is a process of investigating one’s practice to understand and improve it” (p.87). Kemmis’s model of

Action Research was used in which, planning, acting, observing and reflecting was done on the activities of researcher and teaching and further planning was also done on the basis of reflection of the researcher and so on. Hence, based on the above- mentioned grounds, Action Research was selected as a suitable method for this study and Kemmis, McTaggart & Retallick, (2004) model of Action Research was used to plan, act, observe, and reflect.

For data generation, analytical memos, field notes, observation, informal conversation with teachers and students, sample of student’s work was used.

Research Site and Participant

This study was conducted in a cooperative English medium private school in Karachi after informed consent from the concerned authorities. Lower secondary Mathematics classroom was taken according to the nature of research question. Inquiry teaching was applied in a whole classroom instead of using a clinical¹ approach. Though a clinical approach is good in terms of individual's attention to all the students', although, it is in an ideal situation, however in a real situation the whole class is to be taught. This study enabled the researcher to know the issues regarding application of inquiry and students' participation in a large classroom. Inquiry teaching strategy was applied, with a dual role of a teacher-researcher plus primary participant of the study, the secondary research participants were the lower secondary Mathematics class students. Furthermore, the researcher was also helped by a friend who was with him and helped to improve instructional strategies of the researcher through discussions, critique and other related feedbacks outside the classroom. In

addition, the friend used to sit in class of the researcher to provide feedback and to share constructive reflections to enrich data.

Findings and Discussion**Do, Discuss and Observe (D-D-O)****Framework**

In this study, Fink (1999) model of active learning was adapted with some changes and modification. According to Fink (1999), model active learning means doing, discussing and observing. Hence, according to this study active participation means doing, discussing and observing Mathematics. In the following section, some illustrative examples would be shared which portray the general findings. Three lessons had been selected on the ground because they show active engagement of the students in the classroom in terms of doing, discussing and observing Mathematics. In addition, these illustrative examples trace the development of students from initial concept to application of the concept to solve daily life problems.

¹ According to this approach a teacher can select a specific group of students to teach instead of teaching the whole class.

Objectives:

By the end of the lesson students will be able to,

- Share their ideas about Average in a group and with the class
- Explore the process of finding Average in group
- Share the process of investigation with the whole class having following question in mind
 - Meaning of Average
 - Verbalise the process of obtaining Average
 - What did they do to reach this solution?
 - How did they know when to stop the exploration process?

Activity:

What's in a Name?

Students in small group had to work through the task of strips with the names written vertically. They had to arrange the strips from shorter to longer names. They had to cut one letter at a time from the end of the longer names and add them to the ends of the shorter names until all names are of the same length. See appendix-F for detailed lesson plan and group work activity sheet.

Fig.2, Introduction of the Concept of Average

The purpose of the lesson was to introduce the concept of average in the classroom through an open-ended inquiry activity. The students had to work in a group to accomplish the task. Before assigning the group work activity, the researcher asked different questions to check students' previous understanding of the topic 'Average'. The question 'what do you mean by average?' was written on the blackboard and different students' responded to the question differently. They responded as, 'out of the hundred of any anything or quantity' and it was the dominant response in the classroom. By further probing they told the formula for finding average given in the Mathematics

textbook to find average of any given quantity. It indicates, that the responses of the students about average were based on the textbook and what Skemp (1999) termed as instrumental understanding because they were unable to explain the idea beyond what is given in the textbook.

After completion of the activity of the brainstorming, the students were divided into five groups comprising of seven students in each group. The worksheets were distributed in the groups and students were directed to follow the instructions written in the worksheet to accomplish the given task. The students were actively engaged in the group by discussing, doing and

observing Mathematics. The detail is given below.

After receiving the worksheet (WS), the students began to discuss the instructions given in the worksheets. While facilitating the group activity, each group was visited by the researcher to provide assistance to each group. While visiting the groups it was observed by the researcher that in one of the groups, the students were discussing about the process of the activity and it seemed that the students had problems to understand the nature of the activity.

S² 1: Do you know how to solve the question?

S 2: But yar (friend), there is no such question given in the WS as we find in our textbooks?

S 3: We will use the formula to find average?

S 2: Which formula, As I already told you that there is no question in the WS.

S 4: Sir (referring to me) tells us that we have to follow the instructions written in the WS.

S 1: We are talking about questions not instructions.

S 2: (After short silence in the group).....Let's ask.

In chorus: Ok.

(Reflection1, 7/02/2009)

The students were suggested by the researcher to follow the instructions that were written in the WS. Thus, the students began to work on the assigned task.

The students were facing difficulties to follow the instructions written in the worksheet mostly due to language barriers. It was the major hurdle to complete the activity within the given time.

TR³: What is going on dosto (friends)?

S 1: Sir we are confused what to do and how to initiate the task.

TR: I have given you worksheets with instructions and you have to follow those instructions.

S 2: There is no such question given in the worksheet as we work in our class given in our textbook.

S 3: Sir..... (Pause).....actually we are unable to understand the

² Student

³ Teacher Researcher (I am with dual role of Teacher and Researcher)

instructions. Could you please explain it to us?

S 4: Yes sir, please.

(Ref.2, 7/02/2009)

While the researcher was teaching, the Mathematics teacher of the particular class was also present there. In the class while, the students were being facilitated in the process of finding average through an open-ended inquiry activity and the students were discussing in the group, the Mathematics teacher intervened and asked the researcher to control the class.

T⁴: The class is going out of your control.

TR: I do not understand what you mean by out of control.

T: The students are making noise, laughing and hence lack of discipline.

TR: No madam, they are discussing and sharing ideas with each other.

T: You have to tell them how to solve the questions and should check the answers.

TR: The instructions are written in the worksheet and students are following instructions.

(Ref.3, 7/02/2009)

However, in addition with discussion the students were actively engaged through doing Mathematics. According to the reflection of the researcher and the observation of his friend, it revealed that after assigning the task the students were engaged in doing Mathematics such as cutting, pasting, colouring and drawing to accomplish the task. The researcher visited one of the groups while facilitating the groups to complete the assigned task. The students were confused about the nature of activity and it seemed that they had not yet read the instructions written in the worksheet.

TR: What is the problem?

S 1: How to start the activity?

TR: You have to follow the instructions written in the worksheet.

Having said that, the researcher went to another group and after a short break he again visited the same group to check the progress of the students' work.

TR: What about your work progress?

⁴ Teacher (Mathematics teacher of that particular class)

S 2: We have written the name in the graph paper.

TR: Show your work to me?

(Ref.4, 7/02/2009)

According to the nature of the activity and instructions written in the worksheet they had to cut the strips and to paste in the given sheet of the paper. The students did the opposite and cut the whole piece of the graph paper instead of the strips. The researcher intervened in the group activity and started brain storming to clear their concept.

TR: What do you mean by strip?

(Ref.5, 7/02/2009)

None of the students in the group replied about the meaning of the strip, and at last, the researcher explained the meaning and students started to work on the task. It showed that the students were struggling with the English language. After the input from the researcher, they started to cut the strips one by one and put them on the table. The problem now the students were facing was to paste the strips in an order in the given sheet of the paper. The researcher realized the

problem of the students' and came to facilitate them to accomplish the task.

S 1: How to paste the strips in the given sheet sir?

TR: Do you have any idea to write number in an order?

S 1: There are.....

S 2: Yes, there are two ways of writing numbers.

TR: Which two ways?

S 3: Ascending and descending order of writing number.

In addition, the researcher probed the students what they meant by ascending and descending order and they explained it to him.

TR: Which order will you follow to solve the particular problem?

S 2: Ascending order.

TR: Why?

S 2: It is written in the worksheet and we have to paste the strips in an order from shorter to longer strips.

(Ref.6, 7/02/2009)

They pasted the strips in the given sheet of paper in an order from shortest to longest according to the instruction written in the worksheet. The researcher

visited almost each and every group to support the students and found that they were facing difficulty in following the instructions in the worksheet. In addition, they were following the instructions as an order rather to construct Mathematics of their own by making connections between the ideas. It indicates students' poverty in independent thinking and reflection. The students were sharing their ideas with each other in the group like "cut the strip", "cut the paper strip in this way", "this one is short", and "paste the longest strip here in this place". In addition, they communicated the process of doing Mathematics with the whole class like, "we pasted the strip from shortest to longest by following the order of writing numbers according to the rules of mathematics". Furthermore, they shared that they cut the boxes (square box of the graph paper) from the longest strip and pasted in the shorter strip to get average strips. It shows the procedural understanding after going through the process of doing Mathematics in the group.

Moreover, discussion and doing Mathematics was supported by active participation through observing

Mathematics. In the group they were listening when one student used to share their idea with the group and also respecting each other ideas. In one of the group the students cut the whole piece of the graph paper instead of cutting the strips. The researcher went there and asked;

TR: What is this?

S 1: Strip of paper with names.

(Ref.7, 7/02/2009)

After further probing, the students told the researcher that they followed the instruction of one of their class mate and he told them that strip will be that way. Though, the friend misguided the group yet they followed him. This indicates the tolerance and listening of each other views in a group work situation.

In the whole class discussion while questioning, the researcher mostly focused on those students who were not active in terms of doing and discussing Mathematics in the group. However, they were actively observing the classroom activities.

TR: Why you were cutting the boxes of longer strips after pasting in the given sheet of paper

S 1: To get equal size of each strip

TR: Why you want to get equal size

S 2: It was a process to understand the concept of average.

TR: What do you mean by average?

S 3: Equaling, balancing

(Ref.8, 7/02/2009)

The D-D-O Framework-Phase I

The data described shows that group activity provided a platform to do, discuss and observe Mathematics in the classroom. The students were freely discussing ideas with each other which is difficult in the traditional teacher-centered classroom. Most of the students in the teacher centered classroom feel shy to share their ideas with the whole class due to teacher's acceptance of the selective responses. However, in the group inquiry there was no danger of selection or rejection of the ideas. Thus, students were free to share their thoughts with each other and to critique the ideas

in a positive manner as shown in the data (Ref. 1, 7/02/2009).

Furthermore, the students' worked in groups to accomplish the task and in the group discussion was dominant. In the classroom the discussion was not only limited to group work while it was two prong, i.e. discussion in the group and discussion with the whole class. Discussion with the whole class was held after group activity to share ideas about the topic and to share the understanding about the process of learning Mathematics. In this discussion the floor was open to each and every student to share their ideas with the whole class. In addition, it is said that engagement in discussion encourages the peers to experiment new ideas and reexamine old assumptions to gain new understanding and experience (McGregor, 2008).

However, the students were not only engaged in the classroom through discussion but they were also actively participating in the classroom through doing Mathematics to accomplish the assigned task given in the group. That was an opportunity for the learners to play or act with the concrete things to perform the mathematical task and it is

considered as the way of knowing the world through their own activity by doing (Keefer, 2002).

Moreover, while working in the D-D-O platform students were not only working actively on the current problem but were trying to make connections between different mathematical ideas. For instance, while working on the problem of the concept of average they were discussing about writing numbers in an order. Thus, making connections between current mathematical ideas and the ideas learned in the previous classes. Similarly, in the group work the students went through the process of inquiry with open ended task having room for discussion and sharing ideas with each other, thus, becoming an independent and autonomous learner in the process of knowledge construction, thus, constructing their own Mathematical learning. For instance, the responses of the students after group activity were different about average as compared to the responses before the inquiry process as shown in the data (Ref. 8, 7/02/2009). The students' changed responses were caused by the open -ended inquiry process to find average known as procedural understanding (Skemp,

1991). Hence, discussion both in the group and with the whole class was an opportunity for the students to share their ideas with each other thus maximizing active involvement with the task and in the classroom.

According to Fink (1999) active learning model, oral presentation or communication is also considered as doing. In Mathematics education, communication is considered as an essential component to remove confusion from children's mind and to develop conceptual understanding (Pound, 2008). In the D-D-O framework the students were involved in sharing ideas in groups as well as with the whole class as evident from the data described. In addition, according to Fink (1999) model of active learning, active participation through observation means watching and listening to the activities when someone is involved in doing Mathematics. Hence, on the one hand the students were listening and watching the activities while working in groups and on the other hand they were watching and listening to the activities going on in the whole classroom. Pound (2008) argued that, "Learning from other people's ideas occurs in conversation or

through observation of how others go about things” (p.27). Hence, in the above-mentioned classroom environment discussion and observation emerged as a powerful form of active learning.

In addition, grounded in the data depicted in the above pages (Ref.3, 7/02/2009) the teacher who used to observe class of the researcher, created some challenges for him. According to the opinion of friend who was present in the classroom along with the researcher, the open-ended tasks designed for the students were a source of disturbance and lack of discipline in the form of discussion and questioning. It shows the perception of the particular teacher about Mathematics learning which according to them should be teacher-centered, telling the students instead of engaging in open ended task through discussions in group. In addition, it reflects the belief that Mathematics is a body of fixed and objective knowledge which can be transferred from more knowledgeable source (teacher, textbook) to less knowledgeable source (student). Hence, the task of students is to receive the knowledge not to construct.

However, students felt some difficulties to understand the nature of the task. Most of the groups were unable to initiate the task without assistance from the researcher. They were searching for questions in the worksheet like that given in the textbook. Since no such close ended questions were given in the worksheet, hence they were worried and confused as the data revealed (Ref. 1, 7/02/2009).

In this class, the students again had to work in small groups to find mean, median and mode of their home's monthly electricity bill's amount. The researcher had told the students in the previous class to bring their monthly electricity bill's amount so that in the next class, work could be done on them. The researcher divided the students into five groups and distributed blank worksheets and wrote the task on the blackboard.

After receiving the worksheets, the students began to discuss the task in the groups. In one of the groups, the researcher visited the students' assigned writing task to one student and other began to share their ideas with each other to complete the given task.

Objectives:

By the end of the lesson students will be able to,

- Apply the learned concepts of average to find mean, median and mode of electricity bill's amount
- Involve in discussion while investigating the process
- Develop an understanding of the process of finding mean, median and mode
- Solve the given problem
- Describe the process of exploration in group and with the whole class

Activity:

In this class the students' had to apply the learned concepts of the averages to solve the daily life problem. They had to work in small group to find mean, median, and mode of the electricity bill's amount brought from homes. For more detail, please see appendix-G.

Fig. 3, Application of the Concept of Averages Through Utility Bills

S1: Moez (pseudonym) will write and we will help him. S 2: Where are the bill's amounts?	TR: It's up to you, however, go from simple to complex. (Ref.2, 11/02/2009)
All the students shared their amount of bill with the whole group and Moez wrote the entire amount in the given sheet of paper.	S1: Friends, I have written all the seven amounts (as the group comprised of seven students).
Moez: First we should find the mode.	S 2: Very good
S 1: First we should find mean then median and then mode.	S 3: Which amount is coming two or more times.
(Ref.1, 11/02/2009)	S 2: Why, is there any problem?
There seemed to be no consensus in the group to resolve the dispute, the researcher intervened in the group and asked;	S 4: Yes, the problem is we have to find the mode of the given data.
TR: What happened?	S 1: Amount 1200 is repeated three times.
S 3: First we have to find which type of average?	S 3: It means amount 1200 is the mode of our data
	In chorus: Yes.
	S 1: I think now your mood (happy with the result) is ok.

(Ref.3, 11/02/2009)

After completion of the group work, they had to share their result, mean, median and mode of electricity bills, with the whole class. The bill amounts were different from group to group ranging from Rs. 1500 to Rs. 12,000. The whole class discussion was little deviated from the mathematical discussion to general electricity consumption and power crisis in the country.

TR: How to reduce electricity consumption?

S 1: We should switch off unnecessary bulbs, tube lights, fans and other electrical equipments.

TR: Why?

S 2: In this way we will be able to save energy.

S 3: In this way we should get rid of the load shedding.

(Ref.4, 11/02/2009)

Furthermore, the students were not only engaged in discussing Mathematics rather they were actively involved in doing Mathematics. For instance, the students in each group first shared their amount of electricity bills with each

other which were written in their Mathematics notebook. After sharing with each other verbally they wrote the amount in the worksheets. Some of the groups with my assistance and some groups without my assistance wrote the bill's amount in an increasing order of the number (ascending order) to find mean, median and mode. In most of the groups one student from the group was assigned the task of writing and the other group members were helping to complete the task.

S 1: Write the least amount first

S 2: (who has the responsibility of writing the task) but I don't know whose amount is least in number

(Ref.5, 11/02/2009)

One student in the group raised his hand by claiming that his amount would be least of all other member in the group, however, another student sitting near to him claimed that his amount is least as compared to the other who had first shared their amounts. After they began to do the work.

The researcher visited nearly all the groups, they found the mode and median

easily, however, they were facing difficulty in finding the mean which involved a formula. In most of the groups the students were facing difficulty with the symbol Σ (Greek letter Sigma which stands for sum of any given amount).

S 1: How to find out mean of the given amount?

TR: According to your opinion how can we find out mean?

S 2: Sum of any given amount divided by number of the items

TR: So, what is the problem?

S 2: We are confused about the symbol Σ , what is the meaning of this symbol and what is its use in the formula?

(Ref.6, 11/02/2009)

The researcher explained to the students' and they laughed on their own and began to work on it. The entire group completed the doing part before the assigned time.

S 1: Hurrah! We finished the work

S 2: I do not know we are in the right direction or not

S 1: What do you mean by this?

S 3: We have applied the correct formula as we learned in the previous class.

S 2: No doubt, we have applied the formula and followed the procedure but how can we say that our answer is right or wrong?

S 3: Han Yar (Yes friends)

(Ref.7, 11/02/2009)

Moreover, when some of the students were actively doing the task, the other students were actively observing the situation. In the group work situation one student was doing the assigned task and the others were observing him along with sharing their ideas with him. For example, in one of the groups while writing the bills' amount, the students mistakenly repeated one number. Meanwhile, other student who was actively observing the writing task interfered and said that, "whose amounts of bills are equal in number". There were two such cases in which the amount of electricity bills was equal and the students mistakenly had written the number three times. All members of the group suddenly got attentive and checked the amounts written in the

Mathematics notebook. They found no similar cases after checking their bills amount except the two and thus the mistake was corrected. It indicates the importance of active observation to correct the mistakes and subsequently learning from it. In addition, it shows the active involvement of students through observation and its importance to solve the problems.

The Mathematics teacher of the class used to observe the lesson while the researcher was teaching the students. She was observing the activities and looked unhappy with the open-ended task being a believer on text book. Here, the word 'believer on textbook' has been used in the sense that while informal conversation and discussion she repeatedly stressed that, "The task of a good teacher is to follow the textbook strictly".

T: You are not following the textbook

TR: I am teaching those topics what you told me to and I am teaching the concepts I was supposed to cover in this week.

T: Yes, of course you are teaching the concepts but you are not teaching the questions from

the exercise given in the Mathematics textbook.

TR: If students understand the concept they will easily solve any question given in the exercise.

T: According to the syllabus, I have to teach the exercise in this week and if you have left the exercise how can I complete the course?

(Ref.8, 11/02/2009)

It does not mean that the teacher was against the open-ended task and inquiry, rather, she was bound and had to follow the order posed from the top. Therefore, following syllabus and textbook is a compulsion.

At the end of phase two, I held an informal conversation with the focused group of students to check their understanding about the lesson taught in that phase.

TR: what do you mean by mode of a given data?

Imaz: Any number which repeatedly occurs in a given data set.

TR: How can you find out mean and median of a given data?

Rizwan: Median is the mid value of a given data and mean can be found by using mathematical formula (He told the formula to the researcher).

TR: Can you tell me the uses of these concepts in your daily life?

Rizwan and Attiq: We can use it to find out averages of utility bills and mathematics test score.

Imaz and Kashan: We can find averages of our class fellows' age, weight, daily temperature, rainfall, dishes cooked in our houses and so on.

(Ref.9, 11/02/2009)

The D-D-O Framework-Phase II

In this phase of the study, the students showed more interest and familiarity with the open-ended activities as compared to the previous phase. The students applied their learnt concepts of Average taught in the previous class to solve their daily life problems through D-D-O framework. The students enjoyed the class while working on the application of the concept of Averages to find averages of their home electricity bills. They could easily find the mode and median of the electricity bills.

However, in some of the groups the students' faced difficulties to find the mean of the given data by using the formula as evident from the data above (Ref.6, 11/02/2009). After clarification of the meaning of the symbol Σ (Sigma) the students began to work on the task. In addition, while discussing the result in the group the students were not only discussing about the given problem but they were also discussing other daily life issues such as power crisis in the country as revealed from the data given above. It shows that relation of Mathematics with daily life enabled the students to go beyond the classroom and discuss general issues (Ref 4, 11/02/2009). In addition, it indicates that the engagement of the students with daily life problems not only enabled them to discuss the issues but it also enabled the students to become critical thinkers and active citizens of the country.

Furthermore, the students had learnt the concept of Average in the previous class but had not applied the concept to solve the related problems. Hence, it could be said that they had the abstract ideas about average in their mind. This class

provided an opportunity for the students to apply the abstract concept practically to solve the daily life issues. In other words, it could be said that in this class the students were materializing their abstract ideas about averages.

Furthermore, to what extent observation (watching and listening) helped students to learn the concept or idea justified from an informal discussion with the students at the end of this lesson, was the last lesson of second phase before commencing work on phase three. The researcher held an informal conversation with four students to check their understanding about the taught concept in phase two. Four students were selected, Imaz, Kashan who were active in terms of doing and discussing Mathematics, Rizwan, Attiq who were passive in terms of doing and discussing Mathematics. No big difference between the understanding of the students who were passive and who were active in terms of doing Mathematics was noticed. However, one difference was that the students who were active in the class responded more in terms of the application of the concept in their real life as revealed in the data given. It indicated that besides doing and

discussing there are other factors which facilitate the learning process. Thus, It can be said that the focused group of students had learned the concept through observation which is also a powerful form of active learning evident from the data (Ref.9, 11/02/2009).

The purpose of the lesson was to introduce the concept of pie graph construction and to present utility bills through pie graphs as shown in the box. 1.2. An open-ended inquiry activity was designed in groups to introduce the concept in the classroom. In this class, focus was put on one group for data collection while teaching the whole class. In this phase of teaching, the size of group was reduced from seven students to four students because inquiry teaching is considered effective in small groups (Anthony, 1996). In addition, an informal conversation was held with the focused group to check their understanding of the concept taught in the previous phase of the study. Two active students were selected in terms of discussing and doing Mathematics and two passive students were selected in terms of not discussing and doing Mathematics based on experience of the researcher while teaching the class

Objectives:

By the end of the lesson, students will be able to,

- Explain the process of pie graph construction
- Construct a pie graph representing gas, electricity and telephone bills amount
- Identify sectors of the pie graph
- Interpret the data constructed on the pie graph
- Tell possible uses of pie graphs in daily life

Activity:

In this activity, the students had to work in small groups comprising four students. In the first phase of the activity, the students had to share the amount of electricity, gas, and telephone bills in the group. In the next phase, they had to find the mode of the given utility bills. In the last phase, they had to construct a pie graph of the given utility bills. For more details, please see appendix-H.

After receiving the worksheets students began to discuss the task in the group.

Attiq: How to start our work?

Imaz: We should follow the instructions written in the worksheet.

Rizwan: I am unable to understand the instructions?

Kashan: Me and Attiq will read the instructions and try to understand the instruction and you both follow us and do the doing part of the task.

(Ref.1, 14/02/2009)

It seemed that the students were facing difficulty to follow instructions written in the worksheets due to their language barrier. However, they managed to initiate the task on their own. The researcher facilitated the group to follow

the instructions to accomplish the assigned task.

In addition, with language barriers the students were not happy with the nature of the task because it was different from the routine activities in the classroom.

Imaz: It is difficult to understand and to construct a pie graph.

TR: Why do you think so?

Imaz: If you could solve one question on the blackboard, then, it could be easy for us to follow you and solve other related questions.

(Ref.2, 14/02/2009)

After completion of the group activity, the focused group students shared the pie graph in the group. The shaded portion of electricity was greater as compared to

gas and telephone bills. They began to discuss the graph in the group.

Imaz: The graph showed that we consume more electricity as

compared to gas and telephone

Attiq: Yes, the graph tells us so.

Kashan: I went to almost each group and they have a similar finding.

Rizwan: The graph looks very beautiful. I will make one such graph and hang in my bedroom.

(Ref.3, 14/02/2009)

The above discussion showed that in one way they were sharing the findings and in the other they were appreciating pie graph. When the researcher went to the focused group, they told him that they enjoyed the activity and would use the concepts to construct pie graphs of other things.

However, the students were not only involved in the discussion but they were involved in doing Mathematics too. First, they shared their bill's amount with each other and each member had three different types of bill's amount e.g., electricity, telephone and gas bills' amount. In the first instance they faced difficulty on how to handle the situation.

Imaz (a student) began to write and all other members shared their bills and they wrote them in the given sheet of paper.

Imaz: Friends, we have to make one graph and how can we put all these amounts in one graph.

Kashan: One solution is that we should work individually.

Attiq and Rizwan: No, according to the instruction we have to work in a group

(Ref.4, 14/02/2009)

The students were confused because they had not read the instructions in the worksheet carefully. There was clearly written that first they had to find out the mode of the given data from the utility bill's amount. Suddenly, Rizwan cried out, "I have found the solution of the problem while reading the worksheet". He showed the written instruction about finding the mode to other members in the group. Hence, first they found out the mode of each utility bill. They wrote the mode of each utility bill in the given sheet of paper.

The next task was to construct a pie-graph representing each utility bill and for that purpose the students had to

construct a circle. However, there was dispute among the group members about the radius of the circle.

Imaz & Kashan: The radius should be 3 cm.

TR: Why?

Imaz & Kashan: It is easy to manage a small circle.

Attiq and Rizwan: It should be 5 cm.

TR: Why?

Attiq and Rizwan: So that it looks attractive and is visible for all.

(Ref.5, 14/02/2009)

Without wasting time, the researcher instructed the students that at the moment they had to follow him and the circle radius should be 4 cm. They replied in chorus "yes".

They constructed a circle of 4 cm radius and followed the instructions written in the worksheet to construct pie graph. One of the students was assigned the responsibility for drawing and the other members were helping him to construct the pie graph. They were making a scale to distribute the amount of bills in different sectors of the pie graph. After

construction of the pie- graph they shaded the different sectors with different colours through mutual consensus in the group.

Moreover, in addition to discussion and doing Mathematics, engagement through observation had a dominant role in the classroom. For example, while constructing the pie graph the students were confused to handle the data. In the meantime, Rizwan cried out, "He found the solution of the problem". All the members of the group got attentive and were ready to listen to Rizwan. In chorus, "Tell us what you found". Rizwan, "we have to find mode of the given data before construction of the graph and it is written in the worksheet". It meant Rizwan was closely reading the instructions.

Similarly, the other students were actively watching when Imaz was working on the construction of the pie graph. They were helping Imaz to construct the circle and to make different sectors in the circle.

Attiq: We have to shade each sector with different colors.

Rizwan & Kashan: Yes, our graph should be very beautiful.

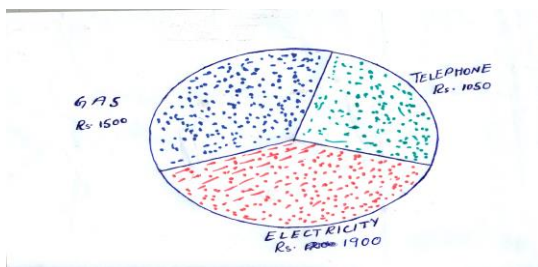
Imaz: Where are the coloured pencils or markers to shade the sectors?

Attiq: Please Rizwan bring coloured markers from the teacher.

Rizwan: Ok.

(Ref.6, 14/02/2009)

FIGURE. 1 Sample of Students' Work Representing Utility Bills through Pie Graph



Note: pen has been traced on students' work so that diagram and words would become legible to readers.

The D-D-O Framework-Phase III

In the above class, to observe the engagement of the students', the researcher focused on the related group and observed their activities very closely. While facilitating the focused group, it was observed that they were discussing with each other to solve the

given task. They were sharing their ideas with each other to perform the task. In addition, they were sharing the amounts of bill with each other and passing different comments on utility bills such as reduce consumption of electricity to avoid load shedding. They were working more independently and autonomously as compared to the previous phases. In addition, as compared to the large group of the second phase the students were very active while working in small groups. Each and every member of the group was involved in inquiry and investigation. Two of the students supposed to be active in the previous class in terms of doing, discussing and questioning were dictating to the passive one in terms of not taking part in discussion, doing and questioning in the previous cycle, were doing the task by drawing and making circle in the sheet provided. They were asking different questions from each other and sharing ideas to accomplish the task, while performing the group activities evident from the data (Ref.1, 14/02/2009).

Moreover, the students were not only involved through doing and discussing Mathematics but they were actively involved through observing (watching

and listening) Mathematics in the classroom. The aspect of listening was more dominant in group discussion and discussion with the whole class. During the discussion in the group, all the students used to listen when one student was sharing his ideas with the group. In addition, respect was given to each member while sharing his point of view with the group. Hence, a social rapport had built among students to share ideas, to listen to each other and to critique each other positively in a more developed form as compared to the previous phases.

The above lesson and activities not only helped the students to relate Mathematics with daily life rather it enabled the students for being actively involved in the process of learning by making connection with the ideas learned in the previous lesson. For instance, grounded from the data while working on the concept of pie graph construction the students also were reinforcing the concept of average learned in the previous class. They found the mode of three different utility bills brought by different students so that they could construct one pie graph representing the whole group. Hence, the

connection between different Mathematical ideas enabled the students to construct pie graph without any confusion. In other words, it can be said that D-D-O framework provided a platform to relate different mathematical ideas and construct new ideas on the basis of the connection of ideas.

Furthermore, the students were very happy with the product and wanted to make the pie graph more attractive and beautiful. They came to the researcher and he provided them three different coloured markers. Due to shortage of time, they only made dots in the different sectors with three different coloured markers instead of shading the whole sectors. In addition, in this class the students worked on their own with little assistance and support from the researcher. They were the real owners of their work. They presented the utility bills very beautifully through pie graph as shown in the figure 1. Furthermore, the students were very happy with their work and left no stone unturned to make the work attractive and beautiful (Ref. 6, 14/02/2009). Moreover, the students' were upset and not happy when they were told to stop working on the task due to time constraints. They wanted to

make the work more colourful and attractive. However, they decided to make a pie graph of their own bills at home individually. The increased interest of students in the task showed ownership of the work and production of their own thoughts and ideas. In addition, the students presented abstract problems based on real life in a pictorial and visual form. Rodrigues (2000) says that pictorial representation helped the students to visualize the abstract concept and interconnection of different ideas. After completion of the activity, the students handed over the work to the researcher, a paper sheet containing pie graphs with different sectors each containing representation of three different utility bills such as electricity, gas and telephone.

Conclusion

The role of the teacher in the study was to facilitate and guide the students in the process of learning. In all the three lessons illustrated previously, students were allowed to work according to their own way and understanding. The students were autonomous to approach the task and they were encouraged by

the researcher for being innovative and imaginative.

It is a fact that students come to class with some ideas based on their prior experiences in and outside the classroom as learner of Mathematics and practices associated with the process of learning. In the present study context these practices were evident in the form of students' way to look at learning Mathematics, nature of Mathematics task and textbook as a source of authentic knowledge as described previously. In the inquiry classroom context, the students after exposing the changed classroom environment, felt that it is difficult to survive without changing their previous conceptions and perceptions. They began to value their own thoughts and experiences to accomplish the assigned task. In addition, the students began to internalize the learning through thinking and reasoning of their own rather to wait for someone to transfer knowledge into their mind. Furthermore, they began to think learning in socio-perspectives and hence, shared their ideas with their peers and also listened to the ideas shared by their peers. It showed that students have to change their prior ways of learning to

involve actively in the inquiry learning process, otherwise, it is difficult to survive in an environment with changed practices.

At the beginning of the study, the students were unable to take decisions about their own learning and the way to approach the problem revealed from the data mentioned in the previous chapter. They were unable to go a single step ahead without the support and decision of the researcher despite having clear written instructions in the worksheet provided to them to work in groups. This again showed the prior experience of the students in the classroom that had no role to make decisions of their own learning, rather, their role have remained as the implementer of the decisions imposed by an external agency.

However, the experience of working through D-D-O framework enabled the students to change their roles in the process of learning. By going through the process of inquiry learning and acquainting with the nature of the task they learned the way to address the open-ended task as described previously. In addition, they saw and felt that their ideas have value and width and they successfully solved the issues or

problems on their own with little external support. Furthermore, it is the D-D-O framework that provided the platform for the students to work independently and autonomously without the external imposition of ideas. Hence, active participation of the students in the Mathematics classroom through doing, discussing and observing enabled the students to become a decision maker in the process of learning and constructing their own Mathematics.

The students in the classroom in addition to active involvement through D-D-O framework were engaged mentally in terms of connection and relation of the Mathematical ideas. The students' while working on open-ended inquiry tasks were linking concepts and ideas to solve the assigned problem as discussed and described in chapter four. For instance, while working on the concept of average they were using the concept of writing numbers in an order to accomplish the task thus making connections between the ideas. In addition, while working on the construction of the pie graph they were using the concepts of average to address the problem as mentioned in chapter four. The connection between

the ideas in one way, was facilitating the students to solve the current mathematical problem and in the other way, it was helping the students to refresh their memory through linkages of the ideas. Hence, it can be said that in the lower secondary Mathematics classroom, inquiry was the major force that enabled the students to make connections between ideas and concepts.

References

- Adler, J. (1997). A participatory inquiry approach and the mediation of mathematical knowledge in a multilingual classroom. *Journal of Mathematics Teacher Education*, 33, 235-258.
- Anderson, R. D. (2002). Reforming science teaching: what research says about inquiry. *Journal of Science Teacher Education*, 13(1), 1-12.
- Anghileri, J. (2006). Scaffolding practices that enhance Mathematics learning. *Journal of Mathematics Teacher Education*, 9, 33-52.
- Anthony, G. (1996). Active learning in a constructivist framework. *Educational Studies in Mathematics*, 31, 349-369.
- Baig, S. & Halai, A. (2006). Learning mathematics rules with reasoning. *Eurasia Journal of Mathematics, Science and Technology Education*, 2 (2), 15-39.
- Barrow, L. H. (2006). A brief history of inquiry: from Dewey to standards. *Journal of Science Teacher Education*, 17 (3), 265-278.
- Baumgartner, E. & Zabin, C. J. (2008). A case study of project-based instruction in the ninth grade: A semester long study of intertidal biodiversity. *Environmental Education Research*, 14 (2), 97-114.
- Basile, C. G. (2002). 'Collecting data outdoors: Making connection to the real world'. In A. R. Teppo and B. Montana

- (Eds.). *Reflecting on NCTM's principles and standards in elementary and middle school Mathematics: Reading from NCTM's school-based journals*. USA: NCTM, 184-188.
- Birenbaum, M. (2002). Assessing self-directed active learning in primary schools. *Assessment in Education*, 9 (1), 119-138.
- Bloom, I. (2007). Extended analyses: Promoting mathematical inquiry with preservice mathematics teachers. *Journal of Mathematics Teacher Education*, 10, 399-403.
- Borasi, R. (1992). *Learning Mathematics through inquiry*. Heinemann Portsmouth NH.
- Carter, A. (2004). Autonomy, inquiry and Mathematics reform. *The constructivist*, 15(1), 1-155.
- Colburn, A. (2000). *An inquiry primer*. California: Department of Science Education State University.
- Corte, E. D. (2007). Learning from instruction: the case of Mathematics. *Learn Inq*, 1, 19-30.
- Costello, P. J. M. (2003). *Action research*. London: continuum
- Dean, B. L. (2009). Action research: In search of an effective teacher professional development strategy. In R. Qureshi. & F. Shamim. (Eds.). *Schools and schooling practices in Pakistan: Lessons for policy and practice*. Oxford University Press.
- Dickens, L. & Watkins, K. (1999). Action research: Rethinking Lewin. *Management learning*, 30 (2), 127-140.
- Dewey, J. (2004). *Democracy and education*. Delhi: Aakra Books.
- Driscoll, M. P. (1994). Jean Piaget's genetic epistemology. *Psychology of learning for instruction*. MA: Allyn and Bacon.

- Ernest, P. (1999). 'Is Mathematics discovered or invented'. *Philosophy of Mathematics Education*, 12, 1-4.
- Ferrance, E. (2000). *Action research*. Northeast and Islands Regional Educational Laboratory at Brown University.
- Fink, L. D. (1999). *Active learning*. Retrieved January 23, 2009 from <http://www.honolulu.hawaii.edu/intranet/activelearning>
- Fogarty, R. (2008). The intelligence-friendly classroom. In Z. P. Barbara, (Eds.). *Teaching for Intelligence (2nd ed.)*. Corwin Press.
- Forster, P. A. (2002). Student participation: an exploration of the possibilities. *Reflective practice*, 3 (2), 125-140.
- Franks, A., & Jewitt, C. (2001). The meaning of action in learning and teaching. *British Educational Research Journal*, 27 (2), 201-118.
- Freire, P. (1972). *Pedagogy of the oppressed*. Penguin education.
- Freire, P. (1998). *Education for critical consciousness*. New York: Continuum.
- Fullan, M. (2001). *Leading in a culture of change*. San Francisco: Jossey-Bass.
- Glover, D. & Law, S. (2002). *Improving learning: Professional practice in secondary schools*. Philadelphia: Open University Press.
- Goos, M. (2004). Learning of Mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, 35(4), 258-291.
- Gulati, S. (2008). Compulsory participation in online discussions: is this constructivism or normalisation of learning? *Innovations in Education and Teaching International*, 45 (2), 183-192.

- Halai, A. (2001). *Role of social interaction in students' learning of Mathematics (in classrooms in Pakistan)*. Unpublished doctoral dissertation submitted to Oxford University, UK.
- Halai, A. (2001). On becoming a 'Professional development teacher': A case from Pakistan. *Mathematics Education Review*, 14, 31-43.
- Halai, A. (1998). Mentor, mentee, and Mathematics: A story of professional development. *Journal of Mathematics Teacher Education*, 1, 295-315.
- Halai, A. (2006). *Ethics in qualitative research: Issues and challenges*. Karachi: Proceedings Multidisciplinary International Conference on Qualitative Research in Developing Countries: Issues and Challenges.
- Halai, A. (2009). Initiating change in Mathematics classrooms. In R. Qureshi. & F. Shamim. (Eds.). *Schools and schooling practices in Pakistan: Lessons for policy practice*. Oxford University Press.
- Halai, N. (2005). Teachers' experience of learning Science: A case of autobiographical reflection. *Science Education International*, 16 (4), 237-242.
- Halai, N. (2006). Learning to use innovative pedagogy: The experience of a primary science teacher in Pakistan. *Science Education International*, 17 (2), 123-132.
- Hogan, K. & Berkowitz, A. R. (2000). Teachers as inquiry learners. *Journal of Science Teacher Education*, 11(1), 1-25.
- Hopkins, D. (1993). *A Teachers guide to classroom research*. Philadelphia: Open University Press.
- Jacobs, K. L. (2005). Investigation of interactive online visual tools for the learning of Mathematics. *International Journal of Mathematics Education in*

Science and Technology, 36 (7), 761-768.

- Jarrett, D. (1997). *Inquiry strategies for science and Mathematics learning*. Northwest Regional Educational Laboratory.
- Jaworski, B. (2006). Theory and practice in Mathematics teaching development: critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, 9 (2), 187-211.
- Jaworski, B. (2007) *Developmental research in Mathematics teaching and learning. Developing learning communities based on inquiry and design*. In Canadian Mathematics Education Study Group Proceedings of 2006 Annual Meeting (pp. 3-16). Calgary, Canada: University of Calgary.
- Joyce, B., Weil, M. & Calhoun, E. (2000). *Models of teaching* (6th ed.). London: Allyn and Bacon.
- Kang, N. H. (2007). Elementary teachers' teaching for conceptual understanding: Learning from action research. *Journal of Science Teacher Education*, 18, 469-495.
- Keefer, M. (2002). Designing reflections on practice: Helping teachers apply cognitive learning principles in an SFT-Inquiry-based. *Interchange*, 33 (4), 395-417.
- Kember, D. (2000). *Action learning and action research: Improving the quality of teaching and learning*. London: Kogan Page.
- Kemmis, S., McTaggart, R. & Retallick, J. (2004). *The action research planner* (Rev. 2nd ed.). Karachi: The Aga Khan University Institute for Educational development.
- Kislenko, K. (in press). *Structuring student's belief in Mathematics: A Norwegian case* proceedings of the MAVI12 workshop, May 2006. Lapland, Finland

- Klein, M. (2004). The premise and promise of inquiry based Mathematics in pre-service teacher education: A poststructuralist analysis. *Asia Pacific Journal of Teacher Education*, 32 (1), 35-47.
- Koshy, N. (2005). *Action research for improving practice*. London: Paul Chapman Publishing.
- Lipson, A., Epstein, A. W., Bras, F. & Hodges, K. (2007). Students' perception of terrascop, a project based freshman learning community. *Journal of Science Education and Technology*, 16(4), 349-364.
- Marr, R. (2005). Teacher beliefs: The impact of the design of constructivist learning environments on instructor epistemologies. *Learning Environments Research*, 8, 135-155.
- McGregor, D. (2007). *Developing thinking; Developing learning: A guide to thinking skills in education*. Open University Press.
- McGregor, D. (2008). The influence of task structure on students' learning process: observations from case studies in secondary school science. *Journal of Curriculum Studies*, 40 (4), 509-540.
- *National curriculum for Mathematics grades I-XII (2006)*. Government of Pakistan Ministry of Education.
- Nicol, C. (1999). Learning to teach Mathematics: Questioning, listening, and responding. *Educational Studies in Mathematics*, 37, 45-66.
- Norton, S. J. & McRobbie, C. J. (2007). Problem solving in a middle school robotics design classroom. *Res Sci Edu*, 37, 261-277.
- Orhun, N. (2007). An investigation into the Mathematics achievement and attitude towards Mathematics

with respect to learning style according to gender. *International Journal of Mathematics Education*, 38 (3), 321-333.

- Orton, A. (1996). *Learning Mathematics issues, theory and classroom practice* (2nd ed.). London: Cassell Wellington house.
- Parker, M. & Hurry, J. (2007). Teachers' use of questioning and modeling comprehension skills in primary classrooms. *Educational Review*, 59 (3), 299-314.
- Piaget, J. (1963). *The origin of intelligence in children*. The Norton library.
- Plowright, D. & Watkins, M. (2004). There are no problems to be solved, only inquiries to be made, in social work education. *Innovations in Education and Teaching International*, 41 (2), 185-206.
- Pound, L. (2008). *Thinking and learning about Mathematics in the early years*. London: Routledge.
- Rarieya, J. (2005). Reflective dialogue: What's in it for teachers? A Pakistan case. *Journal of In-service Education*, 31(2), 313-338.
- Restivo, S., Bendegem, J. P. L. & Fischer, R. (1993). *Maths worlds: philosophical and social studies of Mathematics and Mathematics education* (Eds.). State University of New York Press.
- Riaz, I. (2008). Schools for change: A perspective on school improvement in Pakistan. *Improving Schools*, 11(2), 143-156.
- Rodrigues, S. (2000). 'Pictorial representations: Potential and problems'. *The Mathematics Association of Pakistan: Newsletter*, 6 (1), 4-6.
- Simpson, D. J. & Jackson, M. J. B. (1997). *Educational reform a Deweyan perspective*. New York: garland publishing.

- Skemp, R. R. (1991). *Mathematics in the primary school*. London: Routledge.
- Skerritt, O. Z. (1992). *Action research in higher education: examples and reflections*. London: Kogan page.
- Sousa, D. A. (2008). *How the brain learn Mathematics*. Corwin Press a sage Publication.
- Supovitz, J. A., Mayer, D. P., & Kahle, J. B. (2000). Promoting inquiry based instructional practice: the longitudinal impact of professional development in the context of systematic reform. *Educational policy*, 14 (3), 331-356.
- Sutherland, R. (2007). *Teaching for learning Mathematics*. Open University Press.
- Tan, C. (2008). Improving schools through reflection for teachers: lesson from Singapore. *School Effectiveness and School Improvement*, 19(2), 225-238
- Tee Ng, P. & Tan, C. (2009). Community of practice for teachers: Sensemaking or critical reflective learning? *Reflective practice*, 10 (1), 37-44.
- Their, H. D. & Daviss, B. (2001). *Developing inquiry-based science materials*. New York and London: Teachers College Press.
- Vogrinc, J. & Zuljan, M. V. (2009). Action research in schools-an important factor in teachers' professional development. *Educational studies*, 35 (1), 53-63.
- Walle, J. A. V. (2007). (6th ed.). *Elementary and middle school Mathematics: Teaching developmentally*. New York: PEARSON.
- Watkins, C., Carnell, E. & Lodge, C. (2007). *Effective learning in classrooms*. Paul Chapman Publishing.
- Wells, G. & Wells, G. L. C. (1992). *Constructing knowledge together*. Heinemann Portsmouth, NH.

- Wilhelm, J. A. & Walters, K. L. (2006). Pre-service Mathematics teachers become full participants in inquiry investigations. *International Journal of Mathematics Education in Science and Technology*, 37 (7), 793-804.
- Yang, Y. T. C., Newby, T. J. & Bill, R. L. (2005). Using Socratic questioning to promote critical thinking skills through asynchronous discussion forums in distance learning environment. *The American Journal of Distance Education*, 19 (3), 163-181.

APPENDICES

APPENDIX-A

LESSON OBSERVATION TOOL

Name of School: _____ **Class:** _____ **Date:** _____
Topic: _____ **Teacher:** _____ **Observer:** _____
Children enrolled: ____ **Children present:** ____ **Lesson Start:** ____ **End:** ____

AREA	YES	NO	NOTES
The teachers uses innovative methods to introduce the lesson (e.g. games, story, role-plays)			<i>How the lesson was introduced?</i>
The teacher uses active methods in the lesson. (e.g. songs, games, stories, surveys, pictures, group work, discussions, puppets, drama)			<i>Which active methods? How well are they used?</i>

The teacher uses questions that are directed to all students (e.g. girls and boys, quiet and outspoken, younger and older children).			
The teacher gives children feedback regularly (e.g. acknowledges right answer, probes/prompts for correct answer).			
AREA	YES	NO	NOTES
The teacher provides assistance to all children (e.g. listens to their ideas, asks questions, provides feedback and encouragement).			
The teacher uses innovative methods to conclude the lesson (e.g. quiz, drawings, children's presentations).			<i>How the lesson was concluded?</i>
The teacher has prepared a lesson plan and follows it when teaching the lesson.			

Overall Comments			

APPENDIX-B

Permission Letter to the Principal of the School

To:

Dated: December 31, 2008

Subject: Permission letter for conducting Research Study

Research topic: Teachers' use of inquiry in lower secondary Mathematics classroom

Dear Sir/Madam,

I am a student of Aga Khan University Institute for Educational Development (AKU-IED) and to partially fulfill my M.Ed. course requirement, I have to write a dissertation based on field -work. Therefore, I want to conduct my research study by applying inquiry teaching in lower secondary Mathematics classrooms in your school as per the requirement for action research. The study is supervised by my research supervisor Dr. Anjum Halai.

In this particular study, I am going to apply inquiry teaching strategy in lower secondary Mathematics classroom. The purpose of implementation of this research- based teaching methodology is to see its impact on student active participation in lower secondary Mathematics classroom. The brief description and details about this research study is attached with this letter.

I would visit your school on the period from December 15, 2008 to February 15, 2009. The purpose of the visit would be engaging in the research and data collection.

This would entail teaching, observation and conversations with teachers and students. I would also need to collect some of the students' work. A detailed plan of activities is attached to the letter.

However, the information obtained from the school would not be disclosed under any circumstances. While writing my dissertation, I will use pseudonym of the school instead of using the real name. In addition, I will not disclose the name of my secondary research participants which include students of lower secondary Mathematics classroom. Instead of using the name of the student, I will use pseudonym so that the information remains confidential. The data collected from your school will be kept in a place having no access to anyone except me and my supervisor. Furthermore, after the completion of my dissertation, I will share the findings of my study with you. After completion of my data collection, I will conduct a workshop for the mathematics teachers of your school.

Finally, if you have any questions or queries regarding this research study you can ask me without hesitation and in my absence from the school you can contact me at the address given below.

Yours truly,

Assadullah Khan

C.P M.Ed. 2009

AKU-IED, 1-5/B-VII F.B Area Karimabad Karachi 75950

Email address: assadullah.khan@aku.edu

Cell #.0344-9701127

Enclosed:

- a). Brief description of the study or Project information
- b). Detailed plan of activities

Cc: Supervisor Dr. Anjum Halai

APPENDIX-C

INFORMATION SHEET

Title of the Project	Teachers' use of inquiry teaching in lower secondary Mathematics classroom
Principal Investigator	Assadullah Khan
Designation	C.P, M.Ed. 2009
Institutional address of the investigator	Aga Khan University Institute for Educational Development (AKU-IED) Karimabad Karachi
Email address	assadullah.khan@aku.edu

Purpose of the study

The purpose of the study is to implement inquiry teaching strategy in lower secondary Mathematics classroom to know its impact on student active participation.

Research Procedure

According to the nature of my research question the methodology, I have chosen for this research study is action research. Hence, in this study I am playing dual role of researcher and teacher plus primary participant of this research study. However, my secondary participants will be students of lower secondary Mathematics classroom where, I am going to apply inquiry teaching strategy. In this study, I will use analytic memos, critical friend and field notes to generate data. Therefore, it is requested to let me introduce inquiry teaching with your lower secondary Mathematics class students.

Possible Risks or Discomforts

I am going to apply a teaching strategy in lower secondary Mathematics classrooms which may disturb the routine activities of that particular classroom. The student of that particular classroom may perceive it as an additional burden with other courses going on in their class. However, I will not use students as a means of mere data collection tool rather, I will behave in a friendly way with an environment conducive for

learning and discussion about Mathematics. In addition, I will behave student in way as other teacher of your school behave with the student in normal class routine work.

Possible Benefits

According to the research studies conducted in Pakistan and other educational reports regarding teaching and learning, inquiry teaching is not very much common in Pakistani classrooms. Hence, if your Mathematics teacher of that particular class wants to observe the class than, I have no objection in this regard. In a long run, this study will benefit the students to become active participants in civic affairs.

Privacy, Anonymity and Confidentiality

In any circumstances, I will never disclose the name of your school while writing the research report rather, I will use pseudonyms instead of the real name. Furthermore, I will use the pseudonyms of the students as my secondary research participant instead of using their real names. In addition, the data I have collected from your school will have no access except me and my research supervisor.

APPENDIX-D

THE AGA KHAN UNIVERSITY Institute for Educational Development (AKU-IED) M.Ed Dissertation Class: 2009 Action Research Plan

CYCLE 1

Activities	Beginning/closing date	Details	Duration	Comments
Proposal Development	October 6, 2008 to November 9, 2008	Problem identifying, selecting research methodology and literature review	One month ten days	I will develop research proposal with the help and facilitation of my supervisor
Proposal presentation	November 10, 2008	Presentation	30 minutes	I will present my proposal
ERC work	November 11, 2008 to	Working with ERC forms in	One month twenty days	I will fill the form and submit with

	December 25, 2008	consultation with supervisor		ERC committee
Classroom Observation	January 31, 2009	Classroom observation	One period (45 minutes duration)	I will collect information about teaching and learning in the classroom in addition with children participation
Meeting with teacher	February 4, 2009	Informal discussion about Mathematics teaching	One hour (45 minutes duration)	I will try to find out the way of teaching particular teacher using in the classroom and why
Planning	February 5, 2009 to February 6, 2009	Lesson Planning	Two days	I will prepare lesson plans based on my classroom observation
Teaching session 1	February 7, 2009	Teaching/Collecting data	One period (45 minutes duration)	Memo writing and discussion with critical friend

CYCLE 2

Activities	Beginning/closing date	Details	Duration	Comments
Teaching Session 2	February 9, 2009	Teaching/collecting data	One period (45 minutes duration)	I will write analytic memo based on my teaching and discuss with critical friend
Teaching Session 3	February 11, 2009	Teaching/collecting data	One period (45 minutes duration)	Writing analytic memos and discussion with critical friend
Teaching Session 4	February 12, 2009	Teaching/Collecting data	One period (45 minutes duration)	-do-
Meeting with teacher	February 13, 2009	Informal conversation and discussion	One hour	I will find out information about the

				inquiry teaching strategy I have used in the classroom, its strength and weaknesses. Furthermore, she will give me feedback to improve my teaching strategy in the classroom. This will help me for further planning.
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CYCLE 3

Activities	Beginning/closing date	Details	Duration	Comments
Planning	February 11, 2009	I will plan for further teaching based on the experience of previous teaching	Three days	I will reflect on the teaching and learning in the classroom and design activities for further intervention
Teaching 5	February 14, 2009	Teaching/collecting data	One period (45 minutes duration)	Writing analytic memos on teaching and discussion with critical friend
Teaching 6	February 14, 2009	Teaching/collecting data	One period (45 minutes duration)	Writing analytic memos based on teaching in the classroom and discussion

				with the critical friend
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APPENDIX-E

CODING OF THE DATA AND EMERGING THEMES

Serial No	Codes	Time repeated
1	Contextual understanding	10
2	Meeting with teacher	3
3	Use of Textbook	19
4	Rote learning	7
5	Teacher input in the classroom	5
6	Lack of freedom in the class	4
7	Group inquiry	21
8	Elicitation of children ideas	1
9	Procedural understanding/conceptual understanding	9
10	Student centered class	5
11	Students talk in group	42
12	Students' interest in activities	18
13	Students work in group	26
14	Difficulty to follow the instructions	6
15	Language problem	7
16	Cannot follow the lesson plan	10
17	Examples from daily life	11
18	Students' responses	6
19	Observing	18
20	Questioning	27
21	Teacher views about Mathematics (teaching/learning)	21
22	Students' views about Mathematics	18

EMERGING THEMES:

- Discussing Mathematics
- Doing Mathematics
- Questioning
- Difficulty to follow the instructions
- Teachers Perception
 - Use of Textbook
 - Rote Learning
- Student's Perception
- Classroom
- Teaching and learning
- Students' interest in activities

**APPENDIX-F
LESSON PLAN****Class:** Jr. IV B/F**Subject:** Mathematics**Topic:** Average**Duration:** 45 minutes**Date:** February 9, 2009**Objectives:** By the end of the lesson students will be able to,

- Share their ideas about the average in group and with the whole class
- Explore the process of finding average in group
- Share the process of investigation with the whole class having following question in mind
 - Meaning of average
 - Verbalise the process of obtaining average
 - What did they do to reach this solution?
 - How did they know when to stop the exploration process?

Key Words: Average, Explore

Time	Activities	Content	Resources
5 minutes	Activity # 1 Brain storming	What is average?	Blackboard, chalk
20 minutes	Activity # 2 Exploration	In this activity the students will work in group to explore the process of average. Detailed group activity sheet is attached with the lesson plan	Graph paper, scissors, glue, sheets
15 minutes	Activity # 3 Presentation	In this activity the students will share the process of exploration with the whole class. Evaluation will be conducted embedded with this activity through probing while students presenting their task	Students work
5 minutes	Activity # 4 Home Work	Find the average of the following scores in Mathematics test through the process used while group work, 10, 7, 15, 18, 11, 13 -Each student will bring the amount of electricity bill of their home of the current month to build the next lesson on the data.	

GROUP ACTIVITY SHEET

Activity nature: What's in a Name?

Materials: Graph paper, scissors, glue

Organization: Group

Task:

- Make a list of names of your relatives/ names of group members (At least six names).
- Write the names on the sheet provided/on your notebook-one letter per square. Cut out each name (in strips) and assemble them from shortest to longest.
- Cut one letter at a time from the end of the longer names and add them to the ends of the shorter names until all names are the same length (approximately).
- Glue the “averaged” names on a sheet of paper.
- Write the names of group members in the sheet with your work

Questions for discussion:

- Discuss the meaning of the word average with fellow students
- Can you verbalize the process of obtaining the average?
- What did you do to reach this solution?
- How did you know when to stop cutting the letters?

Note: Please jot down your responses to the above questions, after discussion.

APPENDIX-G

LESSON PLAN

Class: Jr. IV B/F

Subject: Mathematics

Topic: Mean, Median and Mode

Duration: 45 minutes

Date: February 12, 2009

Objectives: By the end of the lesson students will be able to,

- Apply the learned concept of average to find mean, median and mode of electricity bill's amount
- Involve in discussion while investigating the process
- Develop an understanding of the process of finding mean, median and mode
- Solve the given problem
- Describe the process of exploration in group and with the whole class

Key words: Apply, Involve, Develop, Solve, Describe

Time	Activities	Content	Resources
5 minutes	Activity # 1 Brain storming	I will elicit children's ideas by posing high order questions through making bar graph on the blackboard.	Blackboard, chalk
25 minutes	Activity # 2 Group work	I will divide the students in to six groups. In the group the students will find mean, median and mode of their previous month's electricity bill. In the previous class, I had assigned the task to	Worksheets, marker, pencil, erasers, pen

		bring amount of monthly electricity bill of one month and they build the activity on that															
10 minutes	Activity # 3 Group work	Students will find mean, median and mode of the following test scores in Mathematics classroom -12, 15, 16, 10, 12, 18, 19 -66, 70, 58. 66, 75, 62, 80, 66	Note books, pencil, pen														
5 minutes	Activity # 4 Home work	Find mean, median and mode of the heights of the following students of Mathematics class <table><tr><th>No of students</th><th>Height in cm</th></tr><tr><td>1</td><td>55</td></tr><tr><td>2</td><td>50</td></tr><tr><td>3</td><td>66</td></tr><tr><td>4</td><td>55</td></tr><tr><td>5</td><td>77</td></tr><tr><td>6</td><td>66</td></tr></table>	No of students	Height in cm	1	55	2	50	3	66	4	55	5	77	6	66	
No of students	Height in cm																
1	55																
2	50																
3	66																
4	55																
5	77																
6	66																

APPENDIX-H**LESSON PLAN****Class:** Jr. IV B/F**Subject:** Mathematics**Topic:** Construction of Pie/Circle Graph**Duration:** 45 minutes**Date:** February 14, 2009

Objectives: By the end of the lesson students will be able to,

- Explain process of pie graph construction
- Construct a pie graph representing gas, electricity and telephone bills amount
- Identify sectors of pie graph
- Interpret the data constructed on pie graph
- Tell possible uses of pie graph in daily life

Key words: Construct, Identify, Interpret, Sectors

Time	Activities	Content	Resources
5 minutes	Activity # 1 Brain storming	I will ask questions about pie graph to check the previous understanding of the student about the topic	Blackboard, chalk
25 minutes	Activity # 2 Group work	I will divide the students in to ten groups and then distribute worksheets having instructions to perform the task. I facilitate the students while working in group.	Worksheets, Paper sheets, geometry box, data (amount of electricity, gas and telephone bills), calculators
10 minutes	Activity # 3 Presentation	In this activity the students will present their task before the class. I will probe the students through questioning to clarify the process and understanding. Worksheets is attached with the plan	Students work, Blackboard, chalk
5 minutes	Activity # 4 Home work	I will tell student to find mode of the amount of monthly electricity, gas and	

		telephone bills and then construct pie chart of the data and compare the expenditures of the three utility bills	
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GROUP ACTIVITY SHEET

Nature of activity: Construction of Pie graph Gas, Electricity and Telephone bills amounts.

Materials: Paper sheets, data (amount of utility bills), Geometry box, Pencil/pen, erasers

Organization: Group

Task:

- Write the data (amount of gas, electricity and telephone bills separately) in the given sheet in order from smallest to longest
- Find mode of the given data
- Select an appropriate scale according to the nature of the data. For example, Rs. 10 = 1° , Rs. 100 = 1° , then Rs. 100 = 10° or Rs. 1000 = 10° respectively
- Draw circle of any radius in the given sheet of paper
- Measure the angle using protector from your geometry box
- Mark the angles in the circle by using protector
- Draw line from each sector of the circle to the centre of the circle
- Shade each sector using different colours
- The product is the pie graph representing amount of monthly utility bills

Questions for discussion:

- Discuss the process of construction of the graph in your group
- Verbalise the process of the construction

- Write your ideas in the given sheet of papers
- Share your ideas with the whole class

APPENDIX-I

LESSON PLAN**Class:** Jr. IV B/F**Subject:** Mathematics**Topic:** Averages (Mean, Median and Mode)**Duration:** 45 minutes**Date:** February 11, 2009**Objectives:** By the end of the lesson students will be able to,

- Explain average on the basis of the class work assigned in the previous lesson to know the process of average and investigation task assigned as home work
- Develop an understanding of mean, median and mode
- Solve mean, median and mode problems

Key words: Explain, develop, solve, mean, median, mode

Time	Activities	Content	Resources
5 minutes	Activity # 1 Brain storming	In this activity, I will elicit the ideas of the students about average	Blackboard, chalk
15 minutes	Activity # 2 Explanation of mean, median and mode	In this activity, I will explain the concept of mean, median and mode to students. With the help of the below given problem, I will try to explain the three concepts of	Blackboard, chalk

		average. In addition, I will actively involve the students in the activity through questioning													
		<table><tr><th>Roll #</th><th>Maths test score</th></tr><tr><td>1</td><td>15</td></tr><tr><td>2</td><td>10</td></tr><tr><td>3</td><td>10</td></tr><tr><td>4</td><td>5</td></tr><tr><td>5</td><td>10</td></tr></table>	Roll #	Maths test score	1	15	2	10	3	10	4	5	5	10	
Roll #	Maths test score														
1	15														
2	10														
3	10														
4	5														
5	10														
20 minutes	Activity # 3 Group work	I will divide the students in to six groups. The student will find mean, median and mode of monthly electricity bills of their home, with embedded evaluation of students understanding of the topic	Blackboard, chalk, worksheets												
5 minutes	Activity # 4 Home work	-I will tell the students to find mean, median and mode of their monthly gas bills -Bring with you amount of monthly telephone bills.	Blackboard, chalk												

APPENDIX-J**LESSON PLAN****Class:** Jr. IV B/F**Subject:** Mathematics**Topic:** Pie Graph**Duration:** 45 minutes**Date:** February 14, 2009

Objectives: By the end of the lesson students will be able to,

- Develop an understanding of pie graph
- Investigate the process of pie graph formation
- Construct a pie graph
- Explain the process of pie graph formation or construction

Key Words: Pie Graph, Develop, Investigate, construct, Explain

Time	Activities	Content	Resources
5 minutes	Activity # 1 Brain storming	I will elicit the ideas of children about graph and pie graph	Blackboard, chalk
25 minutes	Activity # 2 Group work	In this activity, I will divide the students in to ten groups. In the groups the students will work to understand the process of making pie graph. They will encourage to discuss the process in the group and write it for presentation. Activity worksheets is attached with the plan	Graph paper, sheets, glue, markers, scissors, Activity worksheets
10 minutes	Activity # 3 Presentation	In this activity the students present their task. They will tell the class about the process and their feelings about the process of pie graph formation or construction	Students work
5 minutes	Activity # 4 Home work	In this activity home work will be assigned to students	Blackboard, chalk

		to explore or investigate the following, -Sum of the angles in circle -what do you mean by sectors? -How to plot sectors?	
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GROUP ACTIVITY SHEET

Nature of activity: Formation or construction of Pie chart/Circle graph

Materials: Paper sheet, Graph papers, scissors, tape, glue, pencil, pen

Organization: Group

Task:

- Make bar graph of your names
- Once complete, cut out the bars themselves
- Each bar should be of a different colour
- Tape the two ends of the bars together to form a circle
- Estimate where the centre of the circle is
- Draw lines to the points where different bars meet
- Trace around the full loop using pencil and hand
- The result is a very meaningful "Circle/Pie Graph"

(Walle, 2007)

Questions for Discussion:

- Discuss in you group about the circle graph
- Verbalize the process and write on the paper